2.2. Designing the activities of Pre-service mathematics teachers using Maple to prepare them for the implementation of STEM education

Inna Shyshenko, Tatyana Lukashova, Oleksandr Pypka

***Abstract**. The article is devoted to analyzing and justifying the importance of integrating computer algebra systems (CAS), particularly Maple, into teacher education for pre-service mathematics teachers. The key aspects of utilizing such systems for developing STEM competencies, enhancing research skills, integrating interdisciplinary knowledge, and creating an innovative learning environment are considered. The document highlights the role of Maple in transforming traditional mathematics education into a digital and STEM-oriented paradigm, emphasizing practical approaches to designing educational activities. This study aims to explore the role of Maple-based computer tools in the context of training pre-service mathematics teachers to implement STEM education while studying a discrete mathematics course. The research methods used in the article include theoretical analysis, systematization, and generalization of data from scientific, methodological, and specialized literature. The article addresses the training of pre-service mathematics teachers to implement STEM education by designing their professional activities using the Maple computer mathematics system. The feasibility of integrating Maple into the professional training of teachers as a means of forming digital, research, and interdisciplinary competencies is substantiated. A project activity model is proposed, based on the principles of the STEM approach and utilizing Maple as a tool for modeling, visualization, and analytical processing of mathematical problems. The use of Maple in the training of mathematics teachers promotes the integration of knowledge from related disciplines, characteristic of STEM, and ensures the readiness of future teachers to utilize digital technologies in organizing collaborative learning and project work within the STEM education framework. This aligns with modern trends in mathematics education, which place special emphasis on innovative methods and technologies.*

***Keywords**: STEM education, Maple, pre-service mathematics teacher, digital competence, digital literacy, modeling, project activity.*

DOI: 10.31110/STEM-CaUk-2.2

Introduction

STEM education (Science, Technology, Engineering, and Mathematics) is one of the most important priorities in global education policy, aimed at training qualified personnel for an innovative economy. Global trends clearly indicate an increase in demand for STEM specialists, with forecasts predicting a 20% rise in job opportunities in these areas by 2030. Countries with a high level of development are actively investing in the development of STEM competencies, recognizing that this is a key factor in national competitiveness.

In Ukraine, according to the National Strategy for the Development of Education, the development of STEM competencies is identified as a key area. This involves the formation of students' not only in-depth knowledge of natural sciences and mathematics, but also the development of critical thinking, problem-solving skills, innovative thinking, and interdisciplinary interaction. Traditional education often falls short in preparing young people for the realities of the 21st century, where the ability to apply knowledge in practice and work effectively in a team is crucial. STEM education is designed to overcome these gaps by offering an integrated approach to learning that reflects the complexity and interconnectedness of the real world.

This necessitates adapting teachers to the implementation of STEM education, which requires them not only to possess deep subject knowledge but also to be able to integrate technology into the educational process.

One of the most important aspects of high-quality teacher training is the creation of an effective and dynamic learning environment. Such an environment should rely on interactive models and multimedia resources that enhance student engagement and motivation. Research confirms that the use of dynamic systems enables the development of innovative methodological resources, significantly enhancing the motivation and competence of future teachers for STEM integration (Yurchenko et al., 2022).

In the context of modernizing school education and transitioning to STEM-oriented approaches, the need arises for high-quality training of pre-service mathematics teachers to integrate interdisciplinary learning, research methods, and digital technologies into the educational process (Marchisio et al., 2022). Modern educational challenges require not only a deep understanding of mathematical content but also the ability to utilize modern digital tools, particularly computational mathematics systems, to create models, analyze data, and visualize results. One such powerful tool is Maple, which serves as an interactive platform for mathematical computations and is widely used in higher education to support research and teaching activities.

An analysis of scientific sources on the introduction of digital technologies into the process of professional training of pre-service mathematics teachers (Yunchyk & Fedoniuk, 2019; Cardoso-Espinosa et al., 2021; Gudmundsdottir & Hatlevik, 2018) shows that digital technologies contribute not only to increasing the speed of learning material but also to the formation of deep conceptual understanding among students through visualization and modeling processes, which previously required lengthy analytical calculations (Kovalova et al., 2022).

To ensure high-quality instruction, various computer mathematical systems have been developed, which are divided into several classes. The first class includes systems that work with traditional methods of writing mathematical formulas and are primarily focused on solving complex computational problems and modeling processes, such as Maple, MATLAB, Mathematica, and Maxima. They provide a flexible environment for solving diverse mathematical problems, including discrete structures (Engelbrecht & Borba, 2024). Another class includes dynamic mathematics programs such as GeoGebra and Cabri, which focus on visualization and interactive study of mathematical objects. They are especially useful in the initial stages of training, when clarity and visual perception play an important role (Osypova & Tatochenko, 2021).

Canadian practice in integrating Computer Algebra Systems (CAS), particularly Maple, into STEM teacher training is an important component of modern pedagogical education. Research conducted among 302 Canadian mathematicians showed that the use of CAS in teaching promotes the visualization of mathematical concepts and stimulates students' research activities (Buteau et al., 2013).

Universities such as the University of Waterloo actively incorporate Maple into their curricula, particularly in linear algebra courses, enabling students to gain a deeper understanding of abstract mathematical concepts (Corless et al., 2023).

Additionally, platforms like Möbius provide interactive STEM learning using powerful mathematical engines, which contribute to the development of key competencies in future teachers (Möbius, https://www.digitaled.com/mobius/).

These initiatives demonstrate how integrating CAS into pedagogical practice can significantly improve teacher preparation and support the advancement of STEM education.

Maple (Maple, www.maplesoft.com), as part of the computer mathematics systems, stands out among others due to its extensive mathematical functions for analysis and powerful visualization tools, which make it a versatile tool for teaching discrete mathematics to future bachelor's students. Practical tests and research conducted by us in the process of teaching discrete mathematics to pre-service mathematics teachers demonstrate its effectiveness in improving the quality of specialist training (Lukashova et al., 2022).

Integrating computer algebra systems (CAS) such as Maple into pre-service mathematics teacher education is a highly relevant and promising pedagogical direction.

Research objective. To explore the role of the Maple computer software tools in the context of preparing pre-service mathematics teachers for the implementation of STEM education during the study of the discrete mathematics course.

Research methods. Theoretical analysis, systematization, and generalization of data from scientific, methodological, and specialized literature.

Research Results

Designing activities for future teachers using the Maple software involves creating educational and practical situations in which students utilize digital tools to solve interdisciplinary tasks. This approach enables them to develop not only subject-specific skills but also key competencies, such as critical thinking, analytical skills, creativity, and teamwork. Maple offers numerous opportunities for implementing STEM-related tasks, including graphing, solving equations, studying functions, and modeling physical processes or economic situations mathematically. Such tasks can cover several subject areas, including physics, computer science, and biology, which align with the principles of STEM education.

Maple, like other dynamic mathematical software systems, contributes to the development of several key competencies necessary for a modern teacher, as illustrated in Figure 1.

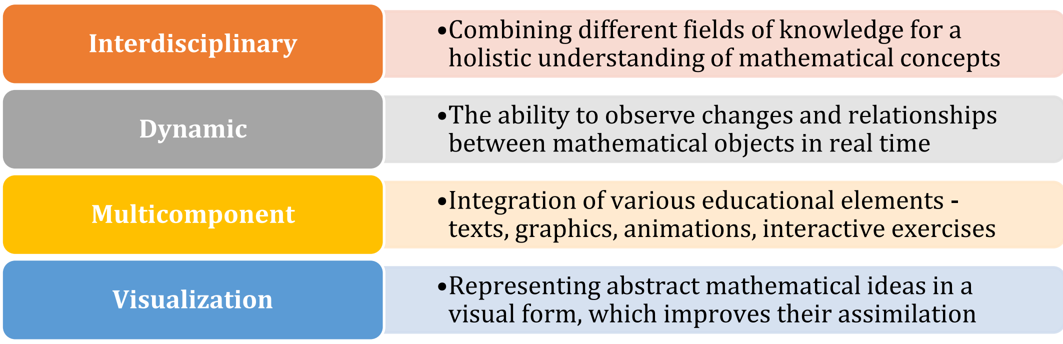

Figure 1. Key competencies developed with Maple

This approach provides a deeper understanding of the material and develops practical skills that are necessary for the successful implementation of STEM approaches in school education.

Maple, as a powerful computer algebra system, offers unique opportunities for the development of key competencies among pre-service mathematics teachers, which are integral to the successful implementation of STEM education. It enables students not only to study theory but also to actively apply it in practice, thereby fostering the development of in-depth research skills, critical thinking, and a creative approach to solving mathematical problems (see Fig. 2).

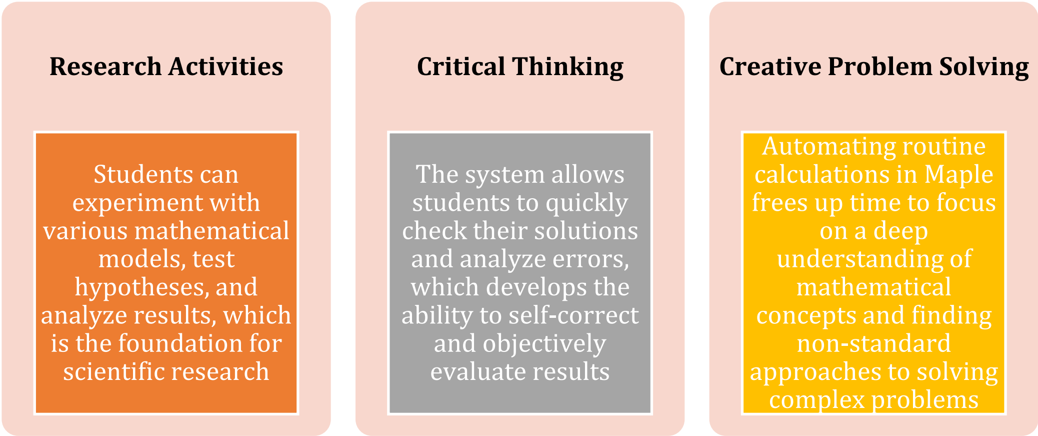

Figure 2. Maple's role in teacher competency development

These aspects are critically important for training teachers who will not only impart knowledge but also inspire students to engage in their own independent exploration and discovery.

In the modern educational environment, the importance of information and digital technologies in the professional training of pre-service mathematics teachers continues to increase annually. The use of computer-based mathematical systems has become an integral part of the educational process, as such tools significantly improve the quality of training, making it more interactive and focused on practical results. At the same time, computer support fully aligns with modern pedagogical standards and existing requirements for specialist training, as it expands the possibilities of pedagogical activity and creates new incentives for creative and experimental work by both students and teachers.

The implementation of STEM education in Ukrainian schools presents several significant challenges for mathematics teachers (Yurchenko et al., 2022). One of the primary issues is the inadequate level of interdisciplinary training among teachers (Namukasa et al., 2022). Many mathematics teachers lack sufficient experience or knowledge in related fields such as physics, chemistry, biology, or engineering, which makes it difficult to integrate these disciplines into a unified educational process. Another challenge lies in the difficulty of integrating abstract mathematics with real-world STEM tasks. Students often perceive mathematics as a set of formulas and theorems detached from real life, without recognizing its practical applications (Zhou & Li, 2021). The teacher's task is to demonstrate how mathematical models help solve specific problems in everyday life, science, and engineering.

Additionally, limited access to modern educational technologies and software, as well as insufficient instructional time for developing practical modeling and simulation skills, pose significant challenges. Addressing these challenges requires a systemic approach to teacher education supported by appropriate tools and resources (see Fig. 3).

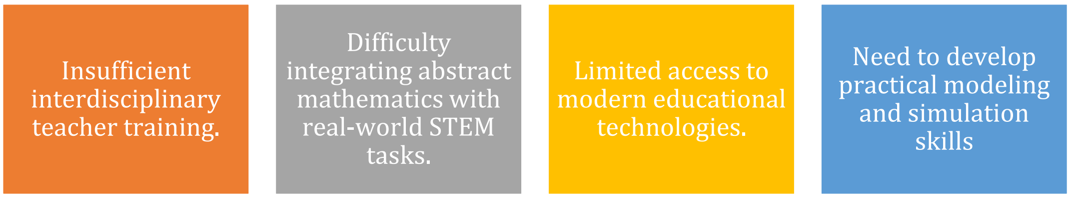

Figure 3. STEM implementation challenges for teachers

The Maple computer algebra system is a powerful and multifunctional tool that can significantly transform the approach to teaching mathematics within STEM-oriented instruction. It provides unique opportunities for training future teachers, allowing them not only to deepen their own mathematical knowledge but also to learn how to effectively utilize modern technologies for modeling and visualizing complex processes.

Maple has powerful computational capabilities that cover symbolic, numerical, and graphical calculations. This enables users to work with algebraic expressions, differential equations, integrals, linear algebra, and many other areas of mathematics with high accuracy and speed. Maple's interactive environment facilitates modeling, simulation, and data visualization, which is critically important for understanding abstract concepts in physics, engineering, and other natural sciences.

In addition, Maple offers specialized packages that are designed to solve problems in physics, engineering, statistics, and chemistry, making it an indispensable tool for interdisciplinary STEM projects. Programming support in Maple enables teachers to create custom applications and interactive models, providing flexibility in developing unique educational materials and tasks. Thanks to Maple, future teachers will be able not only to teach mathematics but also to integrate it into real-world contexts, demonstrating its applications and significance in modern scientific and technological developments.

Maple opens up a wide range of opportunities for integrating mathematics into various STEM disciplines through practical, visual examples. These cases demonstrate how abstract mathematical concepts gain concrete meaning when applied to real-world problems (see Fig. 4).

Physics

- Flight trajectory modeling: analysis of projectile motion taking into account air resistance, allowing visualization of the influence of various parameters on its path.
- Calculation of speed and acceleration: interactive study of kinematics.

Engineering

- Analysis of structural deformations: use of differential equations to calculate stresses and strains in beams, bridges, trusses.
- Optimization of engine parameters: determination of optimal values for maximum efficiency.

Chemistry

- Visualization of chemical reaction kinetics: interactive study of reaction rates and the influence of concentration and temperature.
- Optimization of synthesis conditions: finding the best conditions for maximum product yield.

Statistics

- Processing of real data: analysis of environmental data, sociological surveys, construction of regression models for forecasting.
- Statistical analysis of climate change: studying trends and anomalies.

Biology

- Forecasting population growth: application of logistic models to analyze the dynamics of animal or microorganism populations.
- Modeling of disease spread: simulation of epidemic processes.

Figure 4. Opportunities for STEM integration with Maple

To effectively prepare mathematics teachers to implement STEM education using Maple, a comprehensive methodology has been proposed, which includes several key stages (see Fig. 5). This methodology is based on project-based learning and the integration of theoretical knowledge with practical applications (Kieran, 2019).

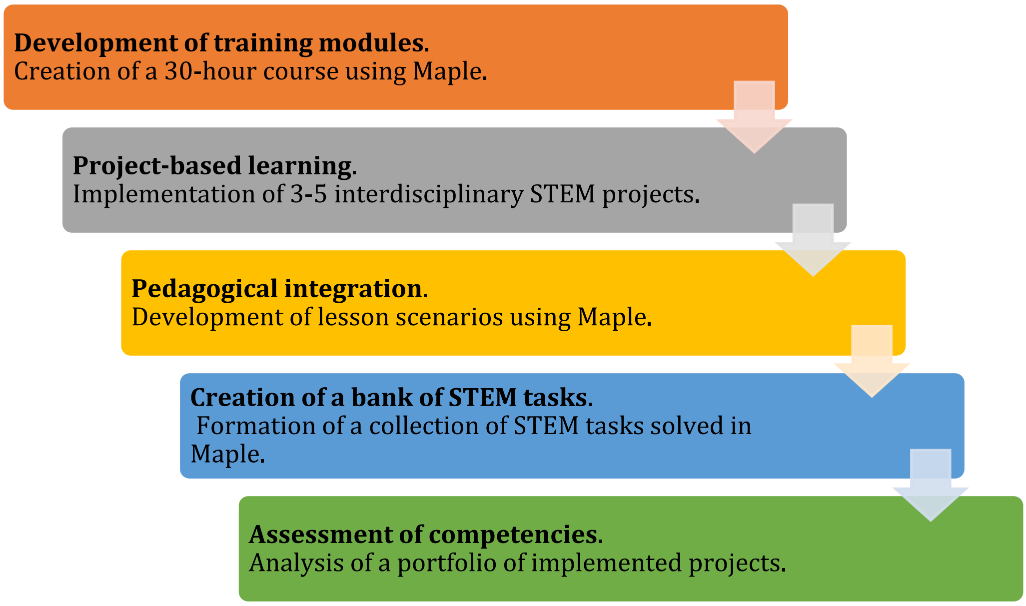

Figure 5. Methodology for STEM training with Maple

The first step is to develop structured learning modules. It is recommended to create an intensive 30-hour course on using Maple, which will cover the program's main functions and their applications in a STEM context.

The second stage focuses on project-based learning. Teachers should complete 3–5 interdisciplinary projects that demonstrate the application of mathematics and Maple to real-world problems in physics, engineering, biology, or chemistry. These projects should be complex enough to require a deep understanding of the material and a creative, analytical approach.

The third stage of pedagogical integration involves the development of lesson scenarios where Maple is used as the primary tool for visualization, modeling, and problem-solving. Teachers should not only learn to use the program but also to effectively implement it within the educational process.

The fourth stage involves creating a bank of STEM problems solved in Maple. This will become a valuable resource for pre-service teachers, encouraging the exchange of experiences.

The final stage involves assessing competencies, which is achieved through a portfolio of implemented projects and a demonstration of the ability to integrate Maple into the educational process.

This approach will ensure deep mastery of the material and confident application of Maple in professional activities.

The implementation of this program for training pre-service mathematics teachers using Maple is expected to yield several significant positive outcomes and benefits, impacting both the teaching staff and the quality of student education.

First of all, teachers are expected to significantly increase their confidence in their ability to effectively integrate STEM approaches into the educational process by approximately 25–30%. This will be achieved by deepening their knowledge in interdisciplinary aspects and mastering modern technological tools such as Maple. This, in turn, will have a positive impact on students' academic performance, as their performance in mathematics and natural sciences is predicted to improve by 10–15%, given that learning becomes more visual, interactive, and practically oriented.

In addition, such training will contribute to the development of key skills in schoolchildren, in particular critical thinking and the ability to solve complex problems, which are the basis of STEM competencies (see Fig. 6). Students' interest in studying STEM subjects will increase significantly as they observe the practical application of the knowledge they have gained. In the long term, this will lead to the formation of an innovative environment in educational institutions, where teachers and students actively use modern technologies for research and creativity, preparing for the challenges of the future.

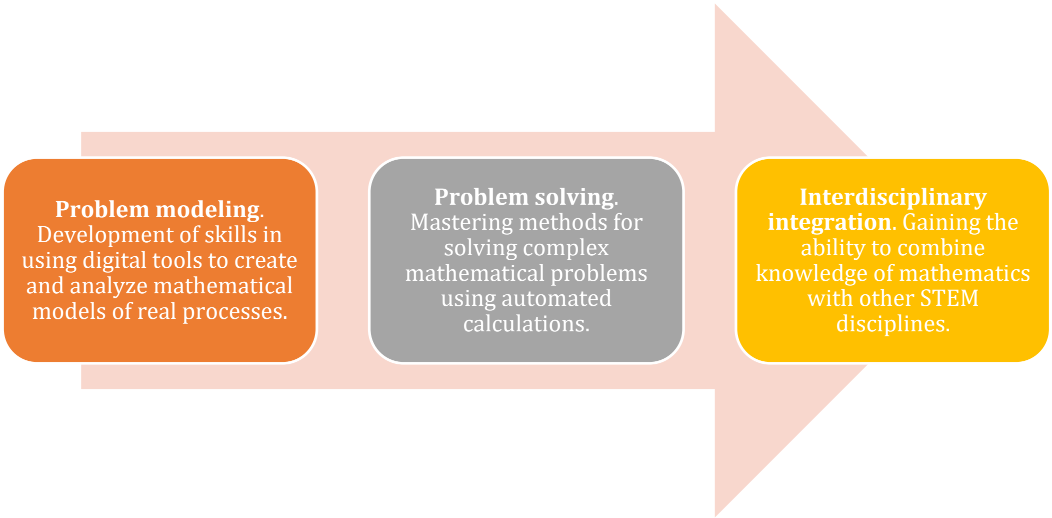

Figure 6. Effects of program implementation

During training, it was found that students who actively used Maple in project activities demonstrated a higher level of digital and professional competence. They are better oriented in the structure of STEM tasks, able to create interactive models, and effectively apply their knowledge in new educational contexts. This is achieved because the content of the presented model is closely related to the formation of methodological readiness of pre-service mathematics teachers for teaching STEM-oriented topics and the integration of digital technologies into educational practice through the following means:

1. Familiarization with the functionality of Maple;
2. Integration of Maple into mathematics courses through educational projects;
3. Development of interdisciplinary STEM tasks using Maple;
4. Independent design by students of educational cases and scenarios for mathematics lessons integrated with natural or technical disciplines;
5. Reflection and analysis of the effectiveness of the tools used.

One of the key advantages of using Maple in training pre-service mathematics teachers is its ability to stimulate the integration of knowledge from related disciplines. This feature is characteristic of STEM education, which requires specialists to take an integrated approach to problem-solving. Maple allows the modeling of phenomena from physics, engineering, economics, and other sciences using mathematical tools that make learning more visual and applied. This ensures the readiness of future teachers to:

– Use digital technologies: effectively use software for demonstrating, analyzing, and modeling various processes;
– Organize collaborative learning: create conditions for students to work as a team on projects that require the integration of knowledge from different subjects;
– Manage project work: supervise student projects aimed at solving real problems using an interdisciplinary approach.

Thus, Maple becomes a tool that not only deepens mathematical knowledge but also develops a broad worldview and the practical skills necessary for implementing innovative educational approaches.

To better understand the benefits of Maple in the context of STEM teacher training, it is useful to conduct a comparative analysis with other popular dynamic mathematics systems (see Table 1). While each system has its own unique capabilities, Maple stands out for its power in symbolic computations and its wide range of functionality.

Thus, Maple is a versatile tool that combines the capabilities of deep mathematical analysis with interactive learning tools, making it an ideal choice for training STEM teachers.

The effective implementation of Maple in the training of pre-service mathematics teachers requires the careful design of educational tasks and methodological materials. These materials should be designed in such a way as to maximize the use of computer technologies, promote the development of interdisciplinary knowledge, and support the formation of STEM competencies:

Table 1. Comparative analysis of Maple with other dynamic mathematical systems in the context of teacher training for STEM education

Characteristic	**Maple**	**GeoGebra**	**Wolfram Alpha**
System Type	Computer Algebra System (CAS)	Dynamic Mathematical System	Computational Knowledge Base
Symbolic Computing	High level	Limited	High level
Numerical Computing	High level	High-level	High level
Graphic Capabilities	Powerful 2D/3D	Powerful 2D/3D (interactive)	Limited (static)
Programming	Embedded language	Scripting	Unavailable
Interdisciplinarity	High (physics, engineering)	Intermediate	High (broad spectrum of knowledge)
Educational Use	Deep research, analysis	Visualization, interactive models	Quick answers, verification

1) computer modeling tasks**:** development of scenarios that involve building mathematical models of real-world phenomena (e.g., population dynamics, motion of bodies) using Maple;

2) problem-oriented cases**:** creation of tasks that require the use of Maple to solve atypical, complex problems that go beyond the scope of standard textbooks;

3) methodological recommendations**:** development of detailed instructions and manuals for students on the use of Maple functionality in pedagogical practice;

4) project work**:** organization of individual and group projects where students use Maple to research interdisciplinary topics.

The key principle should be an interactive and research-based approach that allows students not only to absorb information but also to actively create new knowledge and gain practical experience. This meets the need to transform traditional education toward a digital and STEM-oriented paradigm.

The integration of software, particularly computer algebra systems (CAS) such as Maple, into the training of pre-service mathematics teachers is not only desirable but also a necessary condition for the formation of highly qualified specialists ready for the challenges of STEM education. Maple offers powerful tools for developing research, analytical, and interdisciplinary competencies, which form the foundation of the modern educational process. This enables future teachers to move beyond routine calculations toward a deeper conceptual understanding.

Using Maple in mathematics teacher training facilitates the integration of knowledge from related disciplines, which is characteristic of STEM, and ensures that future teachers are ready to utilize digital technologies in organizing collaborative learning and project work.

Discrete mathematics occupies a special place in the education of pre-service mathematics teachers in the Educational and Professional Program Secondary Education (Physics, Mathematics), at the second (master's) level of higher education, Sumy State Pedagogical University named after A.S. Makarenko (2020). Its content consists of clearly defined structure sets, graphs, combinations, and networks, which differ in their characteristics of discreteness (Lukashova et al., 2022). In the pedagogical context, it is essential that pre-service mathematics teachers possess the skills to visualize and analyze these structures, which enables a deeper understanding of the essence of discrete structures and the mechanisms of their functioning (Lukashova et al., 2022).

Among the sections of discrete mathematics, graph theory is defined as STEM-oriented. Graph theory, as a crucial area of discrete mathematics, has numerous applications in computer science, logistics, bioinformatics, sociology, and other fields, which underscores the need for its in-depth study using modern tools.

Maple has built-in packages that allow users to create, explore, and analyze graphs of various types (directed, undirected, weighted, trees, etc.). In particular, using the GraphTheory package, users can perform operations on graphs, determine their main characteristics (degrees of vertices, number of edges, presence of cycles, connectivity components), perform path searches, and calculate various metric properties (diameter, eccentricity, center, radius, etc.).

The advantage of Maple is its ability to visualize graphs effectively, which significantly facilitates the understanding of abstract concepts and structures. For example, when solving problems to find a minimum spanning tree (Prim's or Kruskal's algorithm), students can observe the step-by-step formation of the tree. Similarly, when studying depth-first search (DFS) or breadth-first search (BFS) algorithms, the implementation in Maple allows users to track the order of visiting vertices and the dynamics of constructing a traversal tree.

Using Maple increases students' motivation to study graph theory and promotes the development of algorithmic thinking, programming skills, and the analysis and synthesis of mathematical models. Additionally, the system facilitates complex calculations with large datasets, significantly reducing time and enabling students to focus on the theoretical analysis of the problem.

Thus, the integration of Maple into the study of graph theory contributes to a deeper mastery of educational material, the development of students' applied competencies, and the formation of a modern scientific worldview. This is especially important in the preparation of future mathematics, computer science, and applied mathematics teachers, for whom the ability to work with computer mathematical systems is a crucial component of their professional activity.

Graph visualization is a central tool for representing discrete structures in Maple. The system supports the construction of graphs as objects consisting of nodes and edges, which can be directed or undirected, depending on the educational tasks. This allows not only for the visual representation of structures but also for their detailed analysis. The Maple GraphTheory package also contains algorithmic tools for determining graph properties, such as vertex degrees, cycles, and connectivity, which are critically important for understanding the topology and functional characteristics of graphs (Maple. URL: www.maplesoft.com/). The relationship between analytical information, calculated using built-in functions, and its graphical display is one of Maple's key advantages. This contributes to the development of a purposeful perception of mathematical objects in students and provides flexible opportunities for research and interpretation of their properties in a visual mode.

One of the main problems of graph theory, typically considered in standard discrete mathematics courses, is the problem of establishing graph isomorphism, which illustrates the algebraic characteristics of graphs. Isomorphism is a key property of a graph, allowing us to determine whether different diagrams (graph drawings) represent the same graph or distinct graphs. This enables further questions about representing a graph in its simplest form or in a way that makes it convenient to establish its properties.

Graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ are called isomorphic if there exists a one-to-one mapping ϕ: $V_1\rightarrow V_2$ that preserves vertex adjacency, i.e., an edge $\{\phi(u),\phi(v)\}\in E_2$ if and only if $\{u,v\}\in E_1$. The fact that graphs G_1 and G_2 are isomorphic is written $G_1\cong G_2$, and the mapping ϕ is called the isomorphism of G_1 onto G_2.

Directly establishing the isomorphism of two graphs "manually" and finding correspondences between their vertices is a rather complicated task. First, it is necessary to check whether the number of vertices and edges in the graphs coincide, and secondly, whether the number of vertices of each degree in both graphs coincides. If at least one of these parameters does not coincide, the graphs are not isomorphic. Otherwise, it is necessary to check whether the isomorphism condition is fulfilled for all correspondences between the vertices of graphs G_1 and G_2 (considering that vertex degrees are preserved during isomorphism).

Let us consider an example of such a task.

Example 1. Prove that the graphs shown in the figure are isomorphic. Solve the problem using the definitions, adjacency, and incidence matrices of the graphs (see Fig. 7).

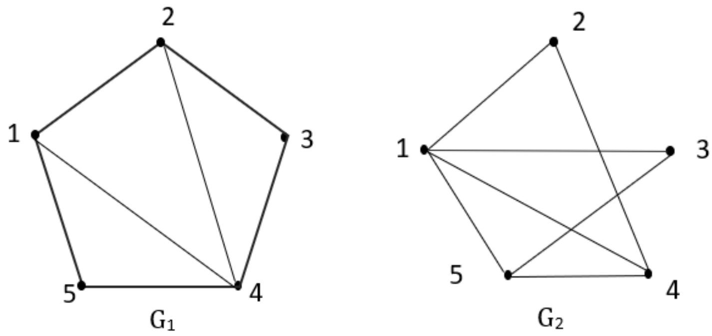

Figure 7. Graph diagrams G_1 and G_2

Solution. It is obvious that the given graphs have the same number of vertices and edges. In addition, in both graphs there is one vertex of degree 4 and two vertices of degrees 2 and 3. Considering that in isomorphism the degrees of the vertices do not change (i.e., vertices of one degree in the first graph correspond to vertices of the same degree in the other graph), the following correspondences between the vertices of the graphs are possible G_1 and G_2:

$$\phi_1 = \binom{12345}{54213},\ \phi_2 = \binom{12345}{45213},\ \phi_3 = \binom{12345}{54321},\ \phi_4 = \binom{12345}{45312},$$

where the first line contains the vertex numbers of the graph G_1, and the second line contains the vertex numbers of the graph G_2.

Let us check whether the correspondence ϕ_1 is an isomorphism of graphs, i.e., let us check whether the condition is fulfilled: $\{\phi_1(u), \phi_1(v)\} \in E_2 \Leftrightarrow \{u, v\} \in E_1$. Let us list all edges of the graph G_1 and find their images under the mapping ϕ_1 :

e_i	{1,2}	{1,4}	{1,5}	{2,3}	{2,4}	{3,4}	{4,5}
$\phi_1(e_i)$	{5,4}	{5,1}	{5,3}	{4,2}	{4,1}	{2,1}	{1,3}

Since the images of all edges belong to the graph G_2, the mapping ϕ_1 is an isomorphism, i.e., cap G sub 1 approximately equal to cap G sub 2, which was to be proved.

Another way to establish an isomorphism is to use adjacency or incidence matrices. This method allows us to reduce the study of graph properties to the study of the properties of their corresponding matrices (adjacency and incidence, respectively).

Recall that the adjacency matrix of a simple graph with n vertices is the matrix $A = \langle a_{ij} \rangle$ of nth order, in which

$$a_{ij} = \begin{cases} 1, \text{if vertices } i \text{ and } j \text{ are adjacent,} \\ 0, \text{otherwise.} \end{cases}$$

Accordingly, the incidence matrix of a graph with *n* vertices and *m* edges is called the matrix B=$\langle b_{ij} \rangle$ of dimension $n \times m$ in which

$$b_{ij} = \begin{cases} 1, \text{if vertex } i \text{ and edge } j \text{ are incident,} \\ 0, \text{otherwise.} \end{cases}$$

Graphs $\mathrm{G_1 = (V_1, E_1)}$ and $\mathrm{G_2 = (V_2, E_2)}$ are isomorphic if and only if they have the same number of vertices and the adjacency matrices of these graphs are similar, i.e. there exists a nondegenerate matrix *Q* such that $A(G_2) = Q \cdot A(G_1) \cdot Q^{-1}$, where $A(G_1)$ and $A(G_2)$ are the adjacency matrices of graphs G_1 and G_2. Or the adjacency matrix $A(G_1)$ of graph G_1 can be obtained from the adjacency matrix $A(G_2)$ of graph G_2 by simultaneous permutations of the same rows and columns.

Graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if and only if they have the same number of vertices and edges, and the incidence matrix $I(G_1)$ of graph G_1 can be obtained from the incidence matrix $I(G_2)$ of graph G_2 by arbitrary permutations of rows and columns.

Let us now prove the isomorphism of graphs G_1 and G_2using their adjacency matrices. It should be noted that establishing isomorphism by proving the similarity of adjacency matrices is a rather cumbersome problem, which is considered in linear algebra. Therefore, it is better to use the second method, which reduces to a simple permutation of the rows and columns of the matrix.

Let us write the adjacency matrix of the graph G_1:

$$A(G_1) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Let's rearrange the first and fourth rows and, respectively, the columns of this matrix, and then the second and fifth rows and, respectively, the columns:

$$A(G_1) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix} = A(G_2).$$

Since, as a result of such transformations, we obtain the adjacency matrix $A(G_2)$ of the graph G_2, then $G_1 \cong G_2$.

The solution to this problem using graph incidence matrices is similar. However, unlike the previous method, arbitrary permutations of the rows and columns of the matrices are used.

Let us now solve the specified problem using the computer mathematics system Maple.

To do this, first connect the GraphTheory module (command > with(GraphTheory):).

We will define each graph with a list of edges and construct graph diagrams in Maple. To do this, we use the commands

> G := Graph({{a, c}, {a, e}, {b, c}, {b, d}, {b, e}, {d, e}}) and > DrawGraph(G) respectively (see Fig. 8).

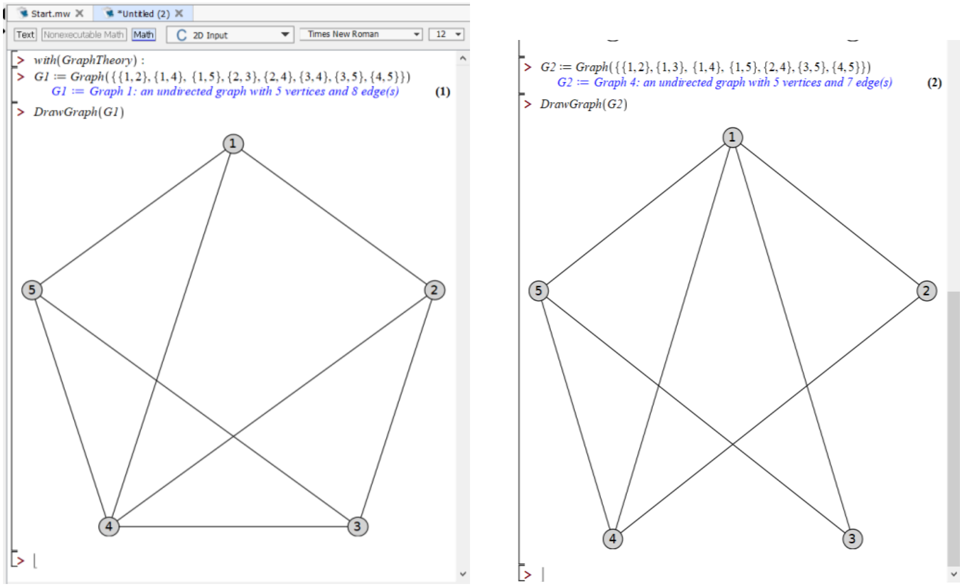

Figure 8. Graph diagrams G_1 and G_2 from Example 1 in Maple

Let's find out if the graphs are isomorphic using the command

> IsIsomorphic(G1, G2, 'mp').

The answer to the question is given in Figure 9 – true, that is, the graphs are isomorphic.

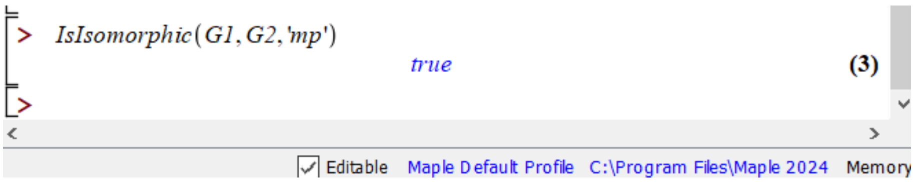

Figure 9. Defining graph isomorphism in Maple

Let's determine the number of vertices of graphs that correspond to each other under isomorphism (command > mp) (see Fig. 10).

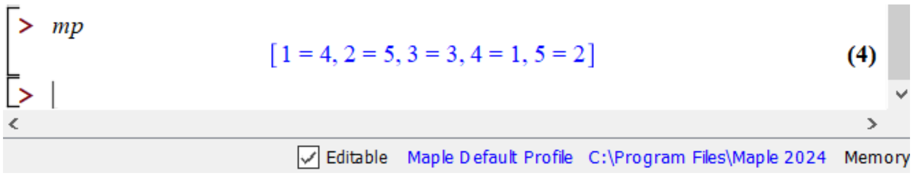

Figure 10. Establishing correspondence between graph vertices under isomorphism in Maple

Thus, Maple allows us not only to determine whether graphs are isomorphic but also to identify a specific correspondence between vertices under the isomorphism. In this case, the system gives a correspondence that corresponds to the vertex substitution $\phi_4 = \binom{12345}{45312}$ given above.

Another way to find out whether graphs are isomorphic is to examine their adjacency matrices. The construction of matrices can also be done in Maple with the command

>A := AdjacencyMatrix(G).

To determine the similarity of adjacency matrices, we will use the LinearAlgebra module and connect it with the command > LinearAlgebra:. Next, we will use the command > IsSimilar(A1, A2) to determine the similarity of the matrices. The answer is true - the matrices are similar, therefore, the graphs are isomorphic (see Fig. 11).

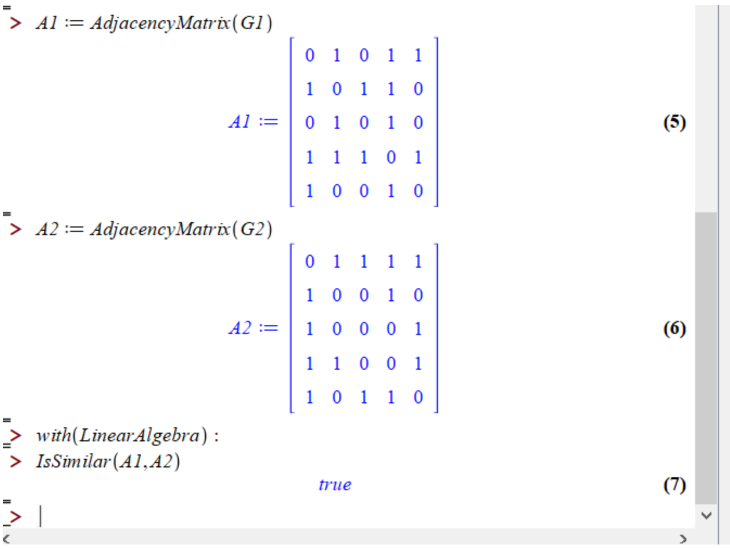

Figure 11. Establishing matrix similarity in Maple

Note that the LinearAlgebra module allows you to determine the similarity of matrices in another way: by finding their eigenvalues. From the course of linear algebra, it is known that matrices are similar if and only if they have the same characteristic roots. Figure 12 shows the result of applying the > eigenvals(A) command, which allows you to find the characteristic roots of a matrix, to the adjacency matrices of given graphs. As shown in Figure 13, the roots are the same; therefore, the matrices are similar, and the corresponding graphs are isomorphic.

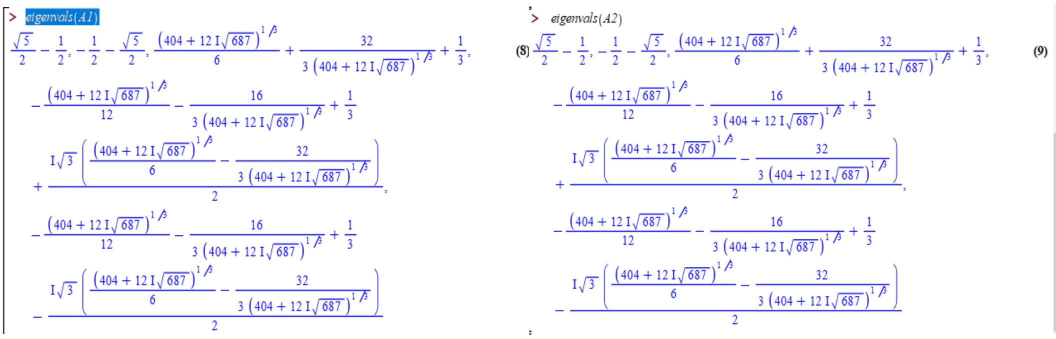

Figure 12. Result of determining characteristic roots of adjacency matrices in Maple

Let's consider another example of using the Maple system to establish graph isomorphism.

Example 2. Find out whether the graph *G*, given by the list of edges E = {{a, c}, {a, e}, {b, c}, {b, d}, {b, e}}, is self-complementary.

Solution. Recall that a graph is called self-complementary if it is isomorphic to its complement. Accordingly, the complement of the graph *G*=(*V*,*E*) is the graph G_1, which has the same set of vertices as the graph *G*, and any two vertices of G_1 are adjacent if and only if they are not adjacent in *G*.

Give the graph *G* and find its complement G_1 (command >G1:= GraphComplement(G)).

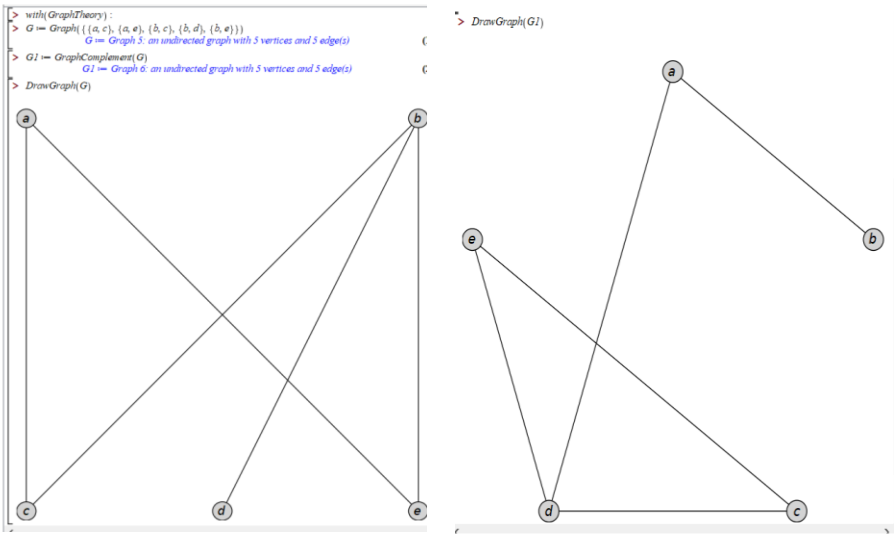

Figure 13. Graph construction and its addition in Maple

Let's determine whether these graphs are isomorphic (theoretically, this is possible, given that the number of edges and vertices in both graphs is the same) (see Fig. 14).

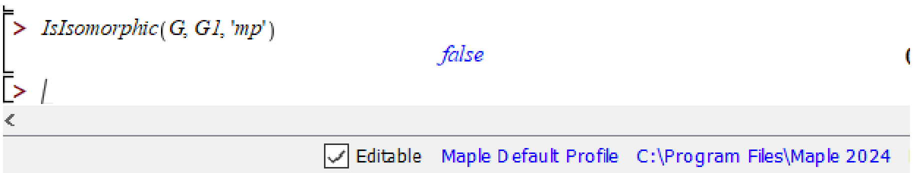

Figure 14. Establishing a graph isomorphism and its complement

Therefore, the graph and its complement are not isomorphic, so this graph is not self-complementary.

One of the key advantages of Maple is the ability to create interactive and dynamic graph visualizations. The system supports real-time modification of graph parameters and displays the effects of these changes, providing students with a clear understanding of internal interactions within the structure. The use of animation enhances clarity when demonstrating algorithms for working with graphs, such as breadth-first or depth-first traversals, cycle detection, or solving routing problems (Kovalova et al., 2022). This enables the creation of experimental training modules, where students can conduct research, test hypotheses, and observe the behavior of mathematical structures in a dynamic manner. Such an approach activates students' creative potential and promotes deeper assimilation of the material.

Along with visualization, Maple provides a comprehensive analytical toolkit for studying the structural characteristics of graphs. The system enables users to calculate various parameters, including vertex degrees, path lengths, and cycles, which are key indicators in graph theory. It also supports the identification of specific types of subgraphs, such as trees or covers, which is useful in teaching discrete mathematics and its applied aspects. Of particular importance is the use of functions for finding optimal routes, minimum covers, and other optimization problems, which allows for the consideration of both static and applied aspects of graph research (Maple, www.maplesoft.com). These capabilities are fundamental for developing practical skills essential for the profession of a mathematics teacher.

Building on the theoretical framework described above, the next section presents empirical evidence from a pilot study conducted with pre-service mathematics teachers. A pilot study was conducted at Sumy State Pedagogical University, enrolling 22 pre-service mathematics teachers in a discrete mathematics course from 2023 to 2025. Participants were divided into a control group (2023-2024, 14 students) (traditional instruction without Maple and without STEM-oriented emphasis) and an experimental group (2024-2025, 8 students) (instruction integrating Maple to implement STEM). In the experimental instruction, Maple was used for modeling, visualization, and solving discrete mathematics problems, as well as for implementing 3–5 project-based tasks that combined mathematics with physics, computer science, or biology to simulate STEM-oriented scenarios. Additionally, Maple was integrated into practical sessions of the discrete mathematics course, allowing students to create interactive models and explore algorithms such as graph traversals and isomorphism.

Comparing the grades of students in both groups for the discrete mathematics course, preliminary conclusions indicate that the experimental group demonstrated significantly higher results. In particular, the average score for solving complex interdisciplinary tasks was 18% higher than in the control group. Surveys were also conducted among the experimental group participants to determine whether the use of Maple and STEM-oriented practices increased their engagement during the course. Eighty-five percent of participants reported confidence in using Maple to solve mathematical problems. Surveys also indicated an increased interest in the course, attributed to its interactive visualization and project-based activities. Furthermore, participants in the experimental group effectively applied mathematical concepts in the contexts of physics and computer science, demonstrating enhanced interdisciplinary STEM thinking skills.

These results suggest that integrating Maple into the training of pre-service mathematics teachers promotes the development of digital, research, and interdisciplinary competencies, fosters critical thinking and creativity, and improves practical problem-solving skills in future STEM educators.

In light of the above, the following recommendations can be formulated to further improve the educational process.

1. Include specialized courses on the use of Maple (or other CAS) for solving applied problems in various STEM disciplines.

2. Develop interactive textbooks and online resources containing examples of using Maple to demonstrate mathematical concepts.

3. Encourage students to carry out research projects using CAS, which will develop their research competencies.

4. Organize training sessions and seminars for teachers on the methodology of integrating Maple into various sections of mathematics and related disciplines.

5. Create interdisciplinary projects that combine mathematics, physics, engineering, and technology, leveraging the capabilities of Maple.

The implementation of these recommendations will contribute to the further transformation of mathematics education in Ukraine, making it more innovative, practically oriented, and aligned with the modern global requirements of STEM education.

Conclusions

Using Maple in the training of pre-service mathematics teachers is an effective tool for developing the key competencies necessary for implementing STEM education. This approach contributes to the modernization of pedagogical education, enriching it with digital tools and fostering readiness for interdisciplinary integration in the educational process. Further research can focus on developing original training modules and methodological recommendations for teachers in higher education institutions specializing in pedagogy.

Designing activities for pre-service mathematics teachers using Maple is an extremely promising direction for modernizing Ukrainian education and adapting it to the requirements of the 21st century. Maple has proven to be an effective tool, allowing not only the deepening of mathematical knowledge but also its integration into the interdisciplinary context of STEM education. A comprehensive training approach, which includes theoretical instruction, project-based learning, and pedagogical integration, ensures the development of all necessary competencies in teachers for the successful implementation of STEM approaches in schools.

Using the Maple computer mathematics system to solve graph theory problems allows the implementation of several didactic objectives: first, to consolidate knowledge of the basic concepts and algorithms used in graph theory; second, to accelerate the solution of complex problems and verify and compare manually obtained results; third, to review material from other mathematical courses (in this case, linear algebra); and finally, to effectively conduct students' research activities in mathematics or other sciences using Maple tools.

In the training of pre-service mathematics teachers, special attention is given to the development of digital competencies, which are essential for the effective use of Maple and similar systems in professional contexts. These competencies encompass not only technical skills in working with CAS, but also an understanding of the methodology behind its application in teaching discrete mathematics, organizing the educational process, and moderating students' learning activities. Teachers should not only master these tools but also act as facilitators who help students maximize the benefits of working with computer tools.

The successful implementation of this program offers significant benefits, including increased professional confidence among teachers, improved academic performance among students, the development of critical thinking and problem-solving skills, and heightened overall interest in STEM disciplines.

In view of this, further prospects include expanding the training program at the national level, covering as many pedagogical universities and postgraduate education institutes as possible. It is also important to conduct long-term studies on the impact of such training on student achievement and their readiness for future careers in STEM fields. This will enable continuous improvement of methods and ensure high-quality education, which is crucial for the innovative development of society.

Designing activities for pre-service mathematics teachers using software, particularly computer algebra systems (CAS) such as Maple, is a relevant direction for preparing teachers to implement STEM education. The use of Maple, like other dynamic mathematical systems, contributes to the development of key competencies in future teachers, particularly the ability to utilize digital technologies for modeling, solving mathematical problems, and integrating interdisciplinary knowledge, which forms the foundation of the STEM approach.

One important aspect of such training is the creation of an effective learning environment based on interactive models and the use of multimedia resources. For example, studies show that dynamic systems, such as GeoGebra AR, can create innovative methodological resources that enhance the motivation and competence of future teachers within STEM, through the interdisciplinary, dynamic, and multi-component nature of the educational process.

Maple, as a computer algebra system, enables students to develop research, critical thinking, and creative problem-solving skills, which are essential for STEM education. It also supports the automation of routine calculations, freeing up time for a deeper understanding of content and the development of pedagogical competencies.

The use of Maple in the training of mathematics teachers promotes the integration of knowledge from related disciplines, characteristic of STEM, and ensures the readiness of future teachers to utilize digital technologies for organizing collaborative learning and project work. This aligns with modern trends in mathematics education, which place special emphasis on innovative methods and technologies.

Thus, the design of activities for pre-service mathematics teachers using Maple should include the creation of educational tasks and methodological materials that stimulate the use of computer technologies in solving complex problems, the development of interdisciplinary knowledge, and the formation of STEM competencies through interactive and research-based approaches. This corresponds to the need to transform traditional education towards a digital and STEM-oriented paradigm.

References

Buteau, C., Jarvis, D., & Lavicza, Z. (2013). On the integration of computer algebra systems (CAS) by Canadian mathematicians: Results of a national survey. *Canadian Journal of Science, Mathematics and Technology Education, 14*(4), 305–320. https://doi.org/10.1080/14926156.2014.874614

Cardoso-Espinosa, E. O., Cortés-Ruiz, J. A., & Zepeda-Hurtado, E. (2021). The development of mathematics and soft skills at the graduate level through project-based learning in times of COVID-19. *TEM Journal*, *10(4),* 1638–1644. https://doi.org/10.18421/TEM104-20

Corless, R. M., Jeffrey, D. J., & Shakoori, A. (2023). Teaching linear algebra in a mechanized mathematical environment. *arXiv*. https://doi.org/10.48550/arXiv.2306.00104

Engelbrecht, J., & Borba, M. (2024). Recent developments in using digital technology in mathematics education. *ZDM Mathematics Education*, *56*, 281–292. https://doi.org/10.1007/s11858-023-01530-2

Gudmundsdottir, G. B., & Hatlevik, O. E. (2018). Newly qualified teachers' professional digital competence: Implications for teacher education. *European Journal of Teacher Education*, *41(2),* 214–231. https://doi.org/10.1080/02619768.2017.1416085

Kieran, C. (2019). Task design frameworks in mathematics education research: An example of a domain-specific frame for algebra learning with technological tools. In *Transactions on Edutainment X*, 265-287. https://doi.org/10.1007/978-3-030-15636-7_12

Kovalova, K., Lysenko, N., & Fedorenko, O. (2022). Zastosuvannia khmarnykh tekhnolohii u protsesi navchannia matematyky [Application of cloud technologies in the process of teaching mathematics.]. *Tekhnolohii elektronnoho navchannia – E-learning technologies*, *6,* 37–44. https://doi.org/10.31865/2709-840062022270265 (in Ukrainian)

Lukashova, T., Drushlyak, M., & Khvorostina, Yu. (2022). Development of pre-service mathematics teachers' soft skills studying the topic "Diophantine Equations". *Physical and Mathematical Education*, *36(4),* 57–63. https://doi.org/10.31110/2413-1571-2022-036-4-008

Maple (n.d.). URL: www.maplesoft.com/products/Maple/index.aspx

Marchisio, M., Remogna, S., Roman, F., & Sacchet, M. (2022). Teaching mathematics to non-mathematics majors through problem solving and new technologies. *Education Sciences*, *12(1),* 34. https://doi.org/10.3390/educsci12010034

Möbius. (n.d.). URL: https://www.digitaled.com/mobius/

Namukasa, I. K., Hughes, J., & Scucuglia, R. (2022). STEAM and critical making in teacher education. In *Handbook of cognitive mathematics* (pp. 939–970). Cham: Springer International Publishing. http://dx.doi.org/10.1007/978-3-030-44982-7_15-1

Osypova, N., & Tatochenko, V. (2021). Improving the learning environment for future mathematics teachers with the use application of the dynamic mathematics system GeoGebra AR. In S. H. Lytvynova & S. O. Semerikov (Eds.), *Proceedings of the 4th International Workshop on Augmented Reality in Education (AREdu, 2021)* (Vol. 2898, pp. 178–196). CEUR Workshop Proceedings. https://doi.org/10.31812/123456789/4628

Semenikhina, O., Yurchenko, K., Shamoniia, V., Khvorostina, Y., Yurchenko, A. (2022). STEM-Education and Features of Its Implementation in Ukraine and the World. Paper presented at the *2022 45th Jubilee International Convention on Information, Communication and Electronic Technology, MIPRO 2022* – Proceedings, 690–695. https://doi.org/10.23919/MIPRO55190.2022.9803620

Semenikhina, O., Yurchenko, A., Ostroha, M., & Shamonia, V. (2024). STEAM education and music: a comparative analysis of practices. *Education. Innovation. Practice*, *12*(9), 78–82. https://doi.org/10.31110/2616-650X-vol12i9-012

Sumy State Pedagogical University. (2020). Educational and professional program: Secondary education (Physics, Mathematics), second (master's) level of higher education. URL: https://sspu.edu.ua/images/2020/doc/opp/osvitnya_programa_serednya_osvita_fizika_matematika_magistr_49bbc.pdf

Yunchyk, V. L., & Fedoniuk, A. A. (2019). Porivnialna kharakterystyka funktsionalnykh mozhlyvostei system komp'iuternoi matematyky v protsesi rozv'iazuvannia zadach [Comparative characteristics of the functional capabilities of computer mathematics systems in the process of solving problems.]. *Informatsiini systemy ta merezhi – Information systems and networks*, *(6),* 90–102. https://doi.org/10.23939/sisn2019.02.090 (in Ukrainian)

Yurchenko, A., Yurchenko, K., Proshkin, V., & Semenikhina, O. (2022). World practices of STEM education implementation: Current problems and results. *International Journal of Research in E-Learning*, *8(2),* 1-20. https://doi.org/10.31261/IJREL.2022.8.2.05

Zhou, C., & Li, Y. (2021). The focus and trend of STEM education research in China : Visual analysis based on CiteSpace. *Open Journal of Social Sciences*, *9*, 168-180. https://doi.org/10.4236/jss.2021.97011