4.2. PECULIARITIES OF TEACHING MATHEMATICS IN CRISIS CONDITIONS (WORK EXPERIENCE OF A UKRAINIAN UNIVERSITY UNDER MARTIAL LAW)

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***Abstract**. Teaching mathematics to students in higher education is a pressing issue in many countries worldwide. A low level of educational motivation and insufficient school preparation cause difficulties in studying mathematics at universities. The situation worsens sharply if crisis conditions are created in the country (pandemics, natural and ecological disasters, military operations). In such situations, students and teachers experience psychological stress, negative emotions, and feelings of anxiety. Ukraine has been at war, but its schools and universities continue to operate. The article describes my own experience of teaching mathematics at the Sumy National Agrarian University (Ukraine) under martial law. An analysis of the educational motivation of students at a higher school in crisis conditions is presented (based on the generalized results of a survey of 90 first-year students), and methods effective in increasing cognitive motivation are described. The article substantiates that the technologies of programmed learning, «flipped classroom,» and visualization of educational material in mathematics are effective in crisis conditions.*

***Keywords**: teaching mathematics; motivation; higher school; crisis conditions; programmed learning; flipped classroom; visualization tools.*

Introduction

Mathematical training of high school students is a significant issue in many countries worldwide. A low level of educational motivation and insufficient school preparation cause difficulties in studying mathematics at universities. The situation worsens sharply if crisis conditions take place in the country (pandemics, natural and ecological disasters, military operations). Students

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and teachers experience psychological stress, negative emotions, and feelings of anxiety in such conditions. Ukraine has been at war, but its schools and universities continue to operate. The article describes our own experience of teaching mathematics at the Sumy National Agrarian University (Ukraine) under martial law.

We analyzed our own experience in two directions: the state of motivation of higher school students in crisis conditions and the organization of mathematics education in universities under martial law.

To determine the level of cognitive motivation among students, a questionnaire was administered. Ninety students from various faculties who study mathematics participated in it. The article describes techniques that are effective for increasing students' cognitive motivation.

The article substantiates that in the conditions of martial law, the scheme of organizing the educational process, which includes three stages «practical lesson – lecture – practical lesson», is effective. It also confirms the effectiveness of the use of elements of the educational technology of the «flipped classroom» in the first stage of this scheme.

The experience of teaching mathematics at the university under martial law confirms the effectiveness of using elements of the educational technology of programmed learning in clarifying the content of the educational material, its structure, and sequence of study, as well as the effectiveness of using visualization tools at the stage of generalization and systematization of the educational material in mathematics.

All the described methodical techniques ensure a sufficient level of assimilation of the mathematics course by university students in crisis conditions and improve the quality of mathematical training of higher school students in normal life conditions.

At the current stage of society's development, schools and universities use various methodical systems of teaching mathematics, which are theoretically grounded and experimentally verified. A schoolteacher and a university teacher

can choose different forms and methods of teaching. Unfortunately, unexpected situations can arise and significantly alter one's usual life. This happened in 2019, when many countries found themselves in the conditions of the COVID-19 pandemic. An unexpected war began in our country on February 24, 2022, and continues to this day. Other countries suffer from natural and environmental disasters (earthquakes, hurricanes, floods, etc.). We call such conditions «crisis». Any country in the world can find itself in such conditions.

Crisis conditions require the state and society to answer the question:

Is it possible and necessary to continue the work of schools, colleges, universities, and other educational institutions?

How to organize the educational process?

What forms and methods of training can be implemented in such conditions?

What learning tools will be effective?

We describe the results of the analysis of our own experience of teaching mathematics for students at the Sumy National Agrarian University (Ukraine) in crisis conditions (martial law has been declared in the country) in the article.

The full-scale invasion of Ukraine began at the end of February 2022. The entire country is divided into different parts: occupied territories, territories in active hostilities, and territories that accept refugees. Our university is located in the Sumy region, which borders an aggressor country. Residential buildings and infrastructure objects are shelled, air strikes are often announced, during which it is necessary to interrupt classes and hide in shelters, and electricity and the Internet are cut off in many areas of the region. At the same time, the country is trying to live, and educational institutions continue to work.

As a result of the analysis of our own experience of work in such conditions, we made the following conclusions:

1. It is necessary to find all opportunities to continue the educational process, because this leads to the stabilization of the psychological state of mind of students and teachers

at the university, even in crisis conditions. Additional efforts by teachers are needed to support students' educational motivation and maintain a positive emotional mood.

2. The educational process is possible only in the form of distance learning (sometimes blended or hybrid learning).
3. In the process of distance learning of mathematics for university students, it is effective to use the elements of the «programmed learning» technology (in terms of content) and the elements of the «flipped classroom» technology (in terms of form).
4. The effectiveness of students' learning increases under the condition of mandatory generalization and systematization of educational material in mathematics with the use of visualization tools.

Research Results

General comments on the structure of the education system in Ukraine

For a better understanding of the article content, we will make a few remarks about the structure of the education system in Ukraine. The education system in Ukraine is multi-level, comprising primary, secondary, and higher education. Secondary school has two levels: basic and senior. High school students choose the profile according to which they continue their studies. Educational programs for classes within profiles differ in the number of hours for studying subjects. Mathematics is a compulsory subject. All (elementary, middle, and high school) students study mathematics.

University students have different mathematics curricula depending on the faculty in which they study. Most of the first-year students at the Sumy National Agrarian University study mathematics. All specialties of the university can be called «non-mathematical» because the mathematics program is small in scope, and it contains only those topics that are necessary for

professional activity or for mastering professional disciplines during further education.

The problem of the mathematical training of pupils and students

Modern society needs specialists with a high level of training. Ukraine, like other industrialized countries, has a technical infrastructure that requires constant maintenance, development, and updates. Insufficient personnel training can lead to a decline in production, the destruction of infrastructure, and a halt in civilizational development, resulting in numerous negative consequences for the Ukrainian people. Mathematics learning has a special place in the training of future specialists who ensure the economic viability. Recently, there has been a decline in the quality of mathematical training of specialists in various areas. The problem is important and relevant. Understanding the relevance of this problem at the state level led to the declaration of the academic year 2020-2021 as the Year of Mathematics Education in Ukraine (Presidential Decree, No. 31/2020). The issue of mathematical training for pupils and students is relevant for many countries worldwide. Scientists have observed a general trend of declining mathematical knowledge among pupils and students, as well as significant difficulties in studying mathematics courses (Petrillo, 2016; Lo, Hew, & Chen, 2017), which also impacts their personal educational motivation.

The problem of educational motivation of students and ways to increase it during the study of mathematics

Ukraine has been living in war conditions for more than a year, but schools and universities continue to work. Students' educational motivation, in particular the motivation to study mathematics, decreases in crisis conditions.

We have investigated the level of educational motivation of students at the Sumy National Agrarian University when studying mathematics under martial law.

We conducted a survey of students of various specialties studying mathematics for this purpose. Ninety full-time students, including 19 from the Faculty of Agricultural Technologies and Nature Management, 18 from the Faculty of Engineering and Technology, and 53 from the Faculty of Economics and Management, participated in the event.

Students answered the following questions:

1. Is mathematical knowledge mandatory for your future professional activity?

Options for answers:

a) Yes, knowledge of mathematics is important and mandatory

b) Yes, knowledge of mathematics is important, but not mandatory

c) No, knowledge of mathematics is not needed

d) Cannot choose

2. Do you have any difficulties while studying the mathematics course?

Options for answers:

a) Yes, mathematics is very complicated and not understandable

b) Yes, mathematics is complicated, but understandable

c) No, mathematics is easy and understandable

d) Cannot choose

3. What is the main reason for these difficulties?

Options for answers:

a) No difficulties, everything is fine

b) Lack of school mathematical preparation

c) Lack of time planned for the mathematical course

d) other reasons

Let us underline that we did not aim for statistical substantiation of the survey results.

The results of the survey are shown in diagrams 1, 2, and 3.

The first diagram illustrates the distribution of students based on their understanding of the need to study mathematics for future professional activities. As we can see, 87% of respondents consider mathematical knowledge to be important. However, most of them (52%) do not consider mathematics to be mandatory. In our opinion, this result suggests that future specialists anticipate that gaps in their mathematical training will not hinder their success in their profession.

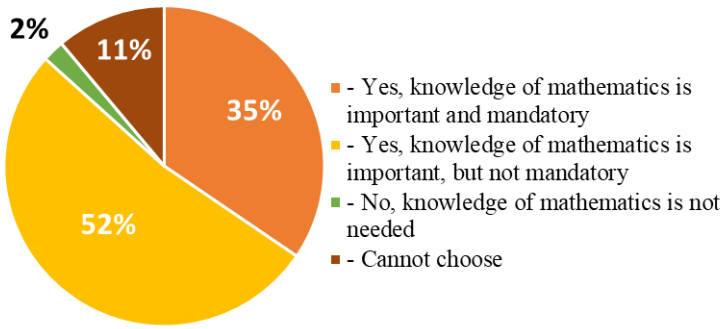


Diagram 1. **The need for mathematical knowledge for future occupations**

The second diagram illustrates students' self-assessment of the difficulties they encounter in studying the mathematics course. Most respondents (80%) believe that the higher mathematics course is difficult, but a bigger part of them (53%) understand the educational material. We were pleased with the results of the survey, which showed the presence of students (unfortunately, only 14%) for whom mathematics is easy and understandable. At the same time, it is surprising that some students are unable to choose a reply to this question. We attribute this to a lack of educational motivation, not only in relation to learning mathematics, but also in relation to learning in general (although it can be explained by other psychological reasons under martial law conditions).

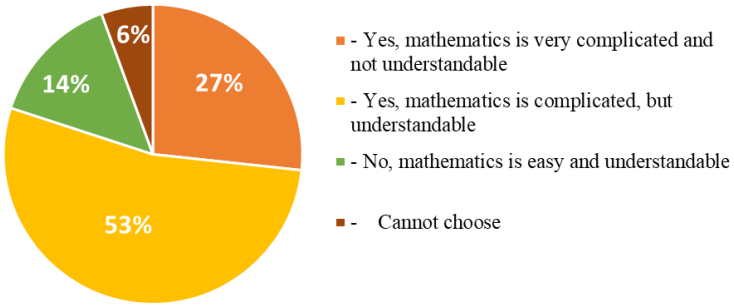


Diagram 2. **The presence of difficulties in studying a mathematics course at the university**

It is evident that studying the higher mathematics course is complicated by students' insufficient knowledge of fundamental concepts in elementary mathematics and their inability to solve typical tasks on various topics in school mathematics. This fact is confirmed by the results of our experimental research. As we can see on the third diagram, almost half of the respondents name gaps in school preparation as the reason for difficulties in studying a higher mathematics course. At the same time, a third of the respondents claim that «everything is fine», although only 14% of students consider mathematics easy and understandable. In our opinion, to explain such results, it is necessary to conduct additional conversations with students to clarify «other reasons», which prevent almost a fifth (21%) of respondents from qualitatively mastering the educational material in mathematics. Perhaps they do not consider low academic results to be a problem for themselves.

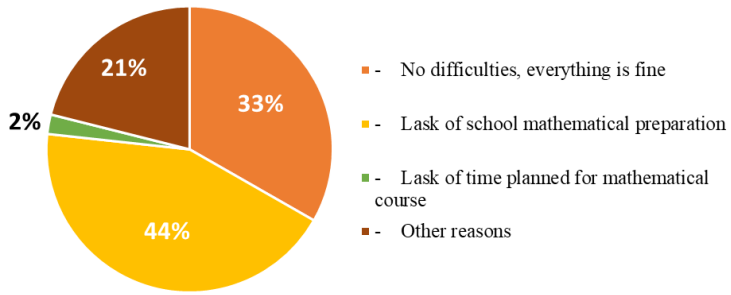


Diagram 3. **Reasons for difficulties in studying a mathematics course at the university**

Summarizing the above, we concluded that:

1. Most students of non-mathematical specialties understand the importance of mathematical training for future professional activities.
2. Mathematics is a rather difficult subject for many students.
3. One of the reasons for difficulties in studying mathematics at the university is the insufficient level of school preparation.
4. Many students have a low level of educational motivation.

In our opinion, the special efforts of university teachers in enhancing student motivation are necessary. A mathematics teacher can reveal the meaning of mathematics in various aspects. When teaching mathematics at Sumy National Agrarian University, we discussed the following aspects with students:

1. The need for mathematical knowledge in everyday life and housekeeping (the ability to calculate, estimate interest on loans, fuel calculations, etc.) for every person.

2. The great importance of mathematics for the development of a person's critical and logical thinking (Zacharopoulos, Sella, Kadosh, 2021).

3. Necessity of mathematical training for specialists of various professions. Industrialized countries rely on technical infrastructure (energy, transport, utilities). This requires engineers and technical personnel who can maintain and improve the vital activity of this infrastructure. Note that an effective technique for improving the motivation to study mathematics is to provide examples of the use of mathematical knowledge in future professional activities.

4. The great importance of mathematics for the defense capability of the state. The modern army requires appropriate technical equipment, a developed military-industrial complex, as well as specialists who can master new technical means. The development of digital technologies necessitates the training of high-level programmers. Therefore, the training of engineers, military personnel, and specialists in the information and digital

sphere is impossible without a solid mathematical foundation.

5. The importance of mathematics for further education and career growth. Mathematics operates with abstract concepts, structures that most people do not use directly in their professional activities. However, today the world is changing at an extremely fast pace. It is becoming increasingly difficult to learn a future profession (and it is even difficult to predict which professions will be in demand in the near future), and professional information is updated very quickly. It becomes impossible to have all the information. Therefore, a person who knows how to distinguish structural abstract elements, establish connections between them, and assess risks will be able to fill these elements with updated content and will be successful and capable in any profession for a long time! Such a person will be able to easily change their field of professional activity if necessary.

In the conditions of the war, we focus students' attention on the importance of mathematical knowledge for the defense capability of the country now and the restoration of the economy (especially the agricultural sector) after it. Our observations suggest that such conversations have a positive impact on students' emotional states, instilling confidence in their future and promoting a sufficient level of educational motivation.

Educational technologies that are effective in the crisis conditions of teaching mathematics

Our experience of teaching mathematical disciplines at the Sumy National Agrarian University under martial law convinces us that only distance (sometimes blended) learning can be implemented in crisis conditions. Let us explain this conclusion.

Some students from our university were evacuated to other cities in Ukraine, while others left for other countries. It is obvious that these students can only study remotely. The rest of the students stay in Sumy city or live in the suburbs, so they have the possibility to visit the university from time to time. We conducted classes remotely or in a blended format (some students were

present in the classroom, while others connected to the class online via Zoom at this time).

In the process of teaching mathematics to SNAU students, we were convinced of the possibility of implementation and the effectiveness of using the elements of «programmed learning» technology (in terms of content), «flipped classroom» technology (in terms of form), and visualization technology.

Use of elements of programmed learning technology

The concept of programmed learning originated a considerable time ago. Modern researchers associate it with the works of the famous American psychologist S. Pressey, who patented special machines for testing students' knowledge in the 20s of the 20th century. The author of the developments believed that the assimilation of knowledge occurs step by step; the transition to learning new material is possible only if correct answers are given to previous questions or the teacher corrects incorrect answers (Pressey, 1926, 1927). The idea of programmed learning received further development in the works of the American psychologist B. Skinner. The scientist claimed that it is important to constantly «reinforce» the student with positive evaluations, stimulation in the learning process, which means encouraging the student with the help of achieving certain success every day in the so-called «short term» (Skinner, 1954, 1957). Therefore, B.F. Skinner suggested dividing educational material into small portions, which should be learned gradually. The idea of programmed learning was developed into a coherent theory in the works of the scientist. This theory was further developed by various scientists, as well as by psychologists and teachers within the Soviet educational system (Kostiuk, 1964; Elkin, 1965; Rozenberh, 1965; Landa, 1967; Mashbyts, 1968; Krutko, 2010). The analysis of scientific studies on the substantiation of the theoretical foundations of programmed learning and its implementation in the schools' practice of this period allows us to draw a conclusion about the effectiveness of this form of education (Ianchenko, 2016).

At the current stage of the pedagogical science, development programmed learning is understood as a system of

theoretical provisions, organizational forms of means of educational work, which foresees mainly indirect program management of the cognitive activity of pupils (students) and specifies their independent assimilation of knowledge, abilities, skills, and mental development of the individual. The term «programming» in relation to the educational process means the creation of programs that manage the educational activities of pupils (students) while solving cognitive tasks. Such programs are called machine learning (learning algorithms).

Methodologists define the following features of training programs that are the basis of programmed training:

1) selection of educational material, its arrangement in a clear, logical sequence;

2) elimination of non-essential information, dividing the material into certain parts;

3) precise and specific instructions for dealing with the tasks that are necessary for learning each portion of the material;

4) tasks for self-assessment (internal feedback), as well as for teachers’ control (external feedback) during the process of students' assimilation of knowledge, skills, and abilities;

5) controlled move from one portion of educational material to another, the next one.

In practice, we relied on the conclusion of scientists that the main, essential characteristics of programmed learning are the presentation of educational material in separate portions and constant two-way communication between the learner and the teacher.

The implementation of programmed training can take place according to the following scheme:

1) the teacher presents the first portion of the educational material, explains it, asks control questions; if the answers are correct, reports a new portion of the educational material;

2) students perceive the first portion of the material accordingly, learn the content, answer the questions, and move on to learning the next one;

3) after students have mastered the entire amount of material on this topic, the teacher asks questions that allow them to summarize and systematize the educational material.

Students can receive educational information directly from the teacher or use programmed manuals.

The material in such a manual can be presented in linear or branched systems.

According to the linear system of material presentation, it is assumed that educational information is divided into small, manageable portions that are easy to learn and comprehend. Each portion is designed to elicit the student's active response and contains hints or instructions that facilitate the search for the correct answer. Every answer should be checked by the student for its correctness. If the answer is incorrect, he continues to look for the correct one and then proceeds to another section of educational information. A linear program eliminates mistakes and brings satisfaction from success. All students learn the same educational material, but at varying paces.

In textbooks built based on a branched system of material presentation, information is also divided into portions. The difference between a branched system and a linear system is that it has several answers to each question, and after learning a portion of information, you need to choose the right answer. If the answer is correct, the student proceeds to learn the next portion of material; if it is incorrect, they also review the previously learned information. Such a textbook does not allow the students to progress further until they complete the task correctly. The focus is not on avoiding errors, but on clarifying and analyzing them (Fitsula, 2009).

The technology of programmed learning emerged a long time ago and is no longer within the scope of interest for modern textbook authors. We did not find special «programmed» mathematics textbooks. Therefore, we utilized our own developments in our work, which were built upon considering the principles of programmed learning. For example, students were offered structured lectures in which separate portions of the educational material were clearly identified.

The specific features of the educational process organization during military operations are the unplanned interruption of classes, dividing them into parts (the need to respond to an alarm signal, turning off the lights, etc.). The learning process becomes «discrete», often occurring in small intervals.

That is why features of programmed training like systematic selection of educational material, placing it in a clear logical sequence; elimination of non-essential information, division of material into certain parts; precise and specific instructions for performing the tasks necessary to learn each portion of the material; tasks for self- assessment, as well as for teacher's control; a controlled transition from one portion of educational material to another, the next one, allows to implement it in the distance form of education in the conditions of martial law.

We claim that the use of elements of programmed learning enhances the effectiveness of distance learning for mathematics students under martial law conditions.

Lectures and practical classes are traditional components of the mathematics learning process. We singled out the following methodological features of conducting a lecture session, taking into consideration the elements of programmed learning:

1. The topic and the main questions of the topic are clearly formulated.
2. Educational material is provided in small portions.
3. Tasks for learning each portion of the theoretical material are formulated (make an outline, write definitions, write down the formulation of theorems, make a corresponding drawing, make a general diagram, or fill in a table, etc.).
4. Students should send reports with the completed task to the teacher for verification (the deadline for submitting the reports is clearly determined by the teacher). No assessment is given after the task check.

We would like to note that the lecture content can be presented sequentially or with a certain branching structure. Learning the theoretical material should be supported by examples of tasks with complete solutions and comments.

During the practical lesson, tasks of various types related to the corresponding topic are planned to be solved. Students are offered tasks of different levels of difficulty (the student themselves chooses the level of task's difficulty) for self-completion. Students have the opportunity to solve problems using samples and comments. Solutions to tasks must be commented.

Use of technology elements in the «flipped classroom»

Today, scientists continue the discussion about the advantages and disadvantages of distance and blended learning, and the search for effective methods of distance learning of mathematics continues (Weurlander, Cohort, Filipsson, 2017; Hartikainen, Rintala, Pylväs, Nokelainen, 2019; Kotsopoulos, Weatherby, Woolford, 2022; Pereira, Gomes, 2022; Stanberry, Payne, 2023).

The technology of the «flipped classroom» is the most popular one. It can be interpreted as a «pedagogical model in which the typical elements of a lecture and homework are swapped. The flipped classroom approach relies on concepts such as active learning. The value of the flipped classroom is in changing the time of a lecture into time for a workshop where students can question lecture content, test their application skills, and interact during hands-on sessions». A large number of articles are devoted to the comparative analysis of the effectiveness of traditional and flipped learning in distance and blended learning in universities (Lo, Hew & Chen, 2017; Lo & Hew, 2020; Fernández-Martín et al., 2020; Fung, Besser & Poon, 2021; Fornons, Palau & Santiago, 2021; Al Mamun, Azad & Boyle, 2022; Sen, 2022). Most authors confirm the conclusion that the technology of the «flipped classroom» is more effective compared to the traditional one in terms of students' success, their social interaction, and educational motivation.

During the last academic year, when organizing mathematics education for students at the Sumy National Agrarian University, we utilized elements of the «flipped classroom» technology. We would like to note that this technology was

implemented not in a comprehensive scheme, but rather partially. Let us comment on this fact.

We believe that the number of hours allocated to the mathematics course at our university is insufficient. However, the course content is very informative. At the Sumy National Agrarian University, in accordance with the mathematics curriculum, students are traditionally introduced to the elements of linear algebra, vector algebra, analytical geometry, differential and integral calculus, probability theory, and mathematical statistics. The named topics contain a large amount of educational material.

According to the curriculum, based on the number of classroom hours, we plan one lecture and two practical classes for studying each topic in the mathematics course at the university. We use the following scheme to organize students' educational activities: a practical lesson, followed by a lecture, and then another practical lesson.

The mathematics course studied by students at our university is based on the school mathematics course and serves as its continuation. We conducted a comparative analysis of the content of the main topics in the mathematics course studied by university students and the content of the school mathematics course. As a result of the analysis, we found that the educational material of the university mathematics course contains approximately 30% to 50% of the basic concepts of school mathematics. We took this fact into account when organizing the educational activities of students, which consist of three main stages. Let us reveal the content of each stage.

First stage – practical lesson No. 1. At the first stage, the «flipped classroom» technology is implemented. The teacher acquaints the students with the topic and the main questions to be considered in advance of the first practical session. These questions correspond to the content of the school mathematics educational material. The teacher formulates tasks for students who independently process educational material based on school textbooks (tasks are sent to students via email) or other available sources of information.

The experience of teaching mathematics under martial law suggests that it is advisable to clearly formulate questions and

indicate the source from which answers should be found when students independently process the educational material of the school mathematics course.

In our work, we use the author's study guide (Rozumenko, 2014), in which the school mathematics course is systematized according to the main content line (appropriate educational material is sent to the student's e-mail together with the tasks, which are sufficient for completing the tasks).

In the first practical lesson, the educational material that the students have learned independently is discussed.

The second stage – a lecture. The teacher reviews the main concepts of the topic, which are already known to the students, and introduces new concepts, using the techniques of analogy and comparison during the lecture. He focuses attention on the interrelationships of the studied concepts, using methods of generalization and systematization of educational material.

The third stage – Practical Lesson No. 2. Students complete exercises of the main types and solve tasks to generalize and systematize the educational material on this topic, as proposed by the teacher in the second practical session.

Under normal conditions, before military operations on our country's territory, we offered students creative tasks that required a certain level of research skills. Students worked on group projects, the results of which were presented at the end of the topic study. The control papers had individual versions of the tasks.

In crisis conditions, according to our observations, most students lack (or have insufficient) motivation to perform such tasks. Therefore, to improve the quality of assimilating mathematical knowledge in crisis conditions, it is advisable to clearly formulate tasks, consider the main types of tasks, and provide examples of solving such tasks. We even offered one version of the test tasks, and after announcing the grades for performance to the students, we conducted a general analysis of errors, providing correct answers (it is impossible to perform such a general analysis by conducting individual tests).

The use of elements of visualization technology in the process of generalization and systematization of knowledge

A specific feature of mathematical concepts is a high level of abstraction. This fact is a cause of difficulties in learning mathematics for many students from various academic backgrounds. One effective means of teaching mathematics, which enables you to improve your understanding of mathematical concepts, is the visualization of educational material.

There are different approaches to the interpretation of the concept of «visualization». Researchers are examining various aspects of this process. For example, visualization as a means of exploring the world (Evagorou, Erduran, & Mäntylä, 2015); visualization as a learning tool (Macnab, Phillips, & Norris, 2012).

The importance of visualizing educational material in the mathematics learning process has been substantiated for a long time. The analysis of the use of visualization tools in learning and teaching mathematics (Bishop, 1989; Presmeg, 2006) showed that the relevance of this problem is increasing in recent years.

Research into the role of visualization in mathematics learning and teaching continues to be conducted. Scientists investigate the influence of visual representations of pupils and students on the ability to solve problems (van Garderen, 2006; Vale & Barbosa, 2023); the use of visualization techniques to increase the level of creativity of students (Cioca & Nerișanu, 2020); the role of visual images in the process of solving problems from the theory of probabilities (Zorzos & Avgerinos, 2023); the use of various visualization tools in the earning process (Buzan & Buzan B., 2019; Lavrenova, 2019). The new stage of visualization research in education is closely tied to the development of information, communication, and digital technologies (Souto, 2014; Ivanova, 2020; Koval & Besklinska, 2020; Semenikhina, 2020; Schoenherr & Schukajlow, 2023).

In our opinion, the use of specialized methods for visualizing educational material is effective at the stage of generalizing and systematizing knowledge. In the process of teaching mathematics in crisis conditions, we consider the stage of generalization and systematization of educational material to be mandatory.

The generalization of knowledge is considered a method of mental and educational activity in didactics. The didactic content of generalization refers to the selection of essential features, characteristics, formation, and formulation of concepts, laws, and ideas within the educational material.

The result of generalizing a certain level of knowledge is its systematization. That is why, usually in didactics, these two processes are connected and considered together. Generalization and systematization of knowledge are effective means of deepening, universalizing, organizing, understanding, and memorizing knowledge. Generalization of knowledge enables the development of problem-solving abilities by transferring the method of action to a whole class of similar problems, which is a primary objective of teaching mathematics.

Our own experience in teaching mathematics has shown that at the stage of generalization and systematization, two means of visualizing the educational material are the most effective:

1) tables, which are used to systematize the learning material of the topic (concepts, their properties, basic facts, formulas, and equations), the section, as well as the entire mathematics course;

2) schemes that help to make connections between concepts, demonstrate the sequence of studying different questions of each topic, and different topics of the entire mathematics course.

Table 1 below shows the system of basic concepts from the topic «Vectors».

Tasks of three levels can be proposed to the students:

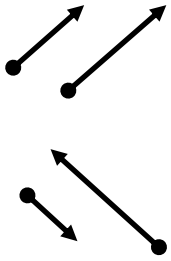
1) make a diagram and/or table on their own (with further discussion in a group or individually);
2) fill in the gaps in the diagram and/or table, if the basis is given (with further comment in the group);
3) comment on the scheme and/or table, which is given in the finished form.

Throughout the entire school year, we have been working under crisis conditions and ensured that the best results are

achieved when the basis of the tables and diagrams is provided by the teacher (in lectures or as a task for independent work).

Table 1.

«Vectors»

Basic concepts *Themes "Vectors"*	*Definition*	*Symbolic designation*	*Drawing*
Vector	Directed segment	$\vec{a}$; $\overrightarrow{AB}$	
Vector module	The length of the segment representing the vector	$\lvert\vec{a}\rvert$; $\lvert\overrightarrow{AB}\rvert$	
The null vector	A vector whose modulus is zero	$\vec{0}$; $\lvert\vec{0}\rvert$	
Collinear vectors	Vectors that lie on the same line or on parallel lines	Same direction $\vec{a} \uparrow\uparrow \vec{b}$ Opposite directions $\vec{a} \uparrow\downarrow \vec{b}$	
Equal vectors	Vectors that have the same direction and equal moduli	$\vec{a} \uparrow\uparrow \vec{b}$ $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert$	

Figures 1 and 2 below show diagrams from the topic «Operations on vectors».

We believe that the basis of such visualization tools should be the same for all students, and the content and structure should be clearly defined and consistent. This contributes to students' awareness of the logical relationships between the main facts of the educational material, allowing them to avoid potential mistakes in the process of assimilation and systematization.

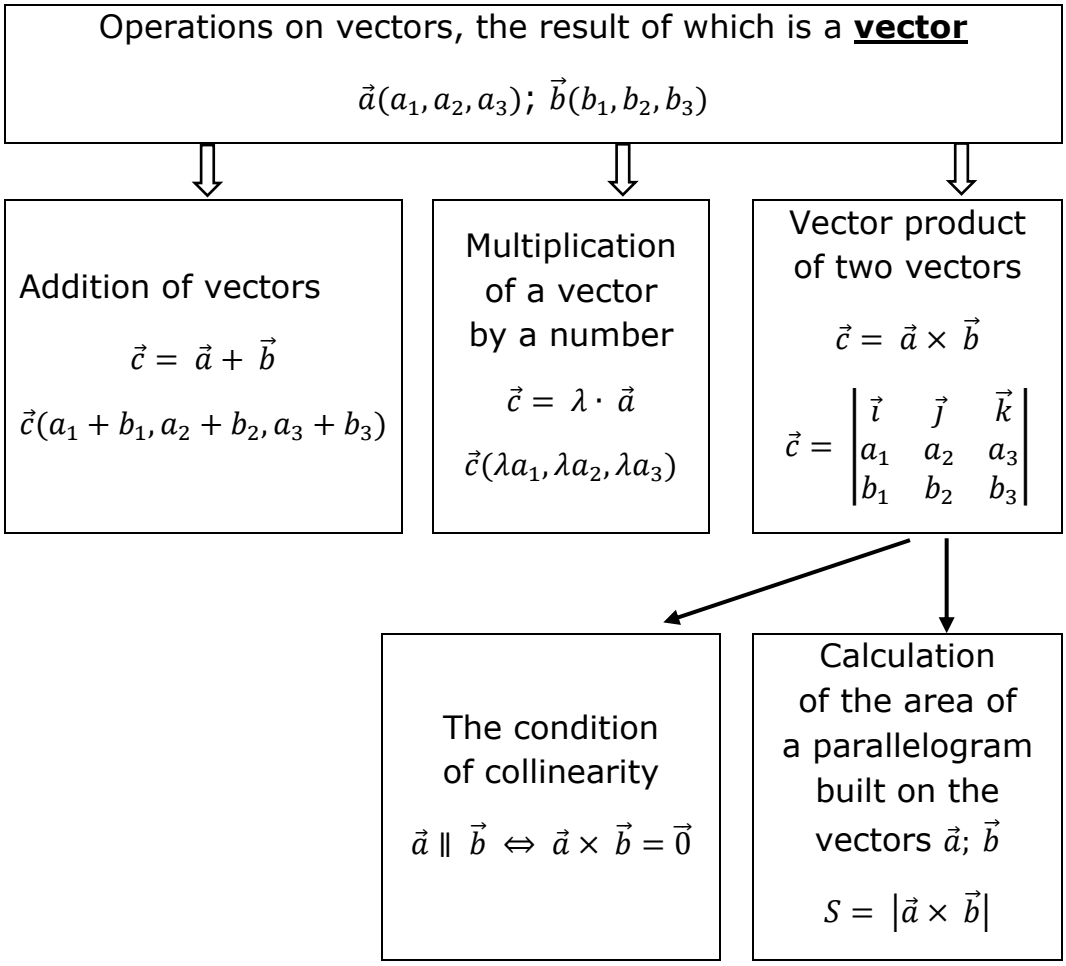


Figure 1. **Scheme 1 «Actions on vectors»**

Students are given the task to independently fill in the gaps, separate logical blocks, establish connections between concepts, and show them with arrows. During the practical lesson, a discussion takes place, and students comment on the completed tables and diagrams. The teacher then demonstrates his version of the tables and diagrams.

Students' independent processing of the means of sign-symbol visualization, along with their further discussion, contributes to their awareness of the main concepts of each topic, the logical connections between concepts, and the main content parts of the educational material.

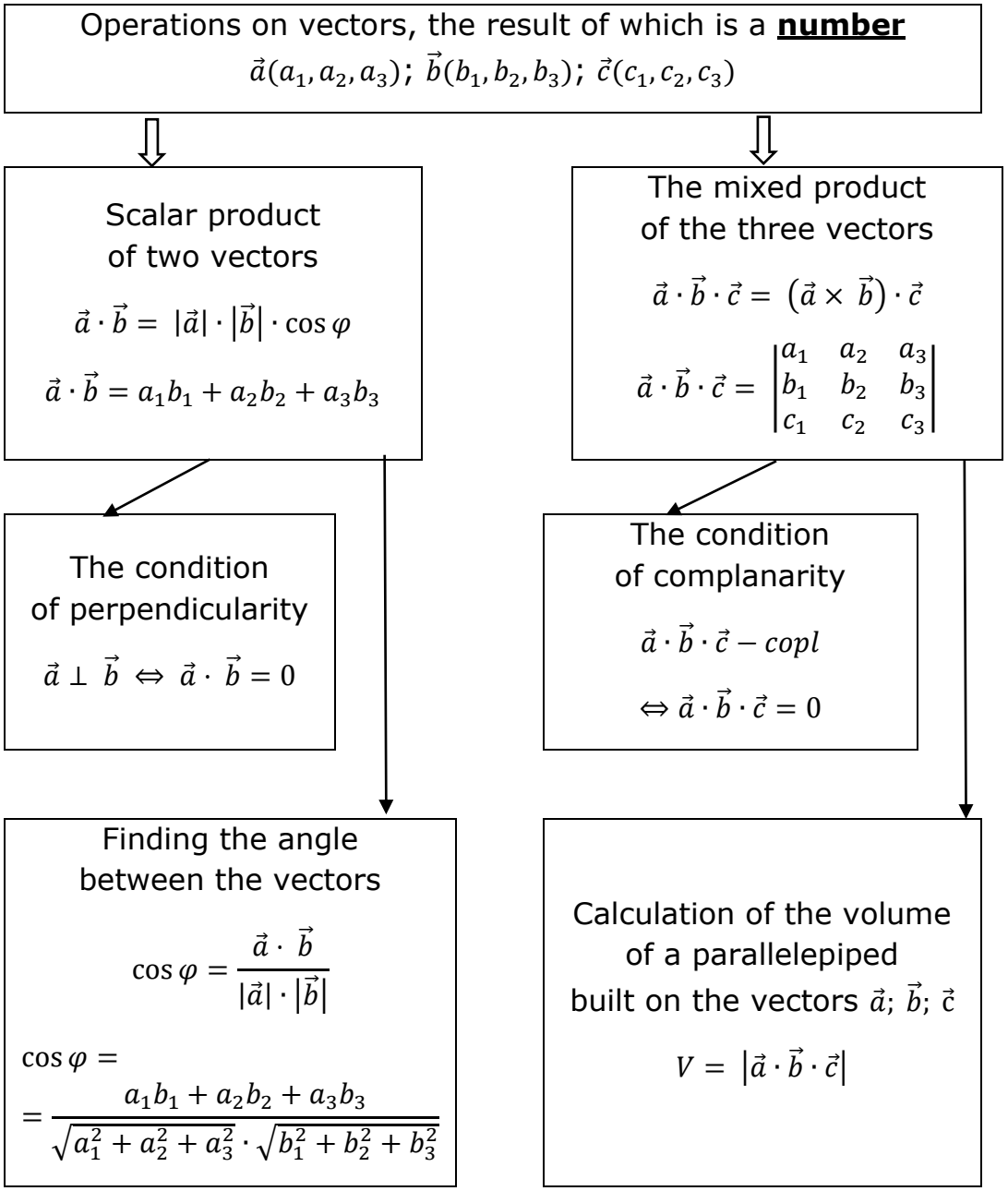


Figure 2. **Scheme 2 «Actions on vectors»**

In our opinion, students should not be required to clearly formulate definitions and properties, nor should they be expected to know formulas. We emphasize that the student should demonstrate the ability to apply a known fact to the solution of a certain type of problem (at the same time, students can use diagrams, tables, lecture notes, as well as notebooks for practical classes), doing control tasks.

Conclusions

The primary tasks of higher education are to develop students' scientific worldview, enhance their competencies, and foster a system of knowledge and skills that enable future specialists to perform their professional activities effectively and continue learning throughout their lives. That is why questions about what to teach and how to teach are always relevant.

Ukraine is a young and independent state. The education system in our country is aligned with European standards. Traditional methodical systems are being improved in universities, and the search for new ones, oriented towards the achievements of modern science, continues.

On February 24, 2022, the war began, and life changed dramatically. This terrible reality has changed the conditions of study, the educational motivation of pupils and students, and the working conditions of schoolteachers and university professors. We call such conditions a crisis. Countries suffering from natural or ecological disasters may also find themselves in crisis conditions. Our article highlights the problems that arise in universities operating under crisis conditions.

The experience of teaching mathematics at the Sumy National Agrarian University during martial law helped us to reach the following conclusions.

Educational motivation of students

The cognitive motivation of students sharply decreases in crisis conditions, and a feeling of anxiety appears, although the understanding of the need to continue studying remains. The teacher can increase learning motivation by discussing the importance of mathematical training for future professional activities and for further education.

Teachers of mathematical disciplines at universities in different countries note the low level of mathematical training of students, especially in non-mathematical specialties, who study in "normal" conditions. One of the reasons for this problem is a lack of motivation for learning mathematics. Therefore, the teacher's

special work is necessary, aimed at familiarizing students with the stages of development of various mathematical theories and the current state of mathematization in science and production.

Educational technologies

Studying mathematics can cause difficulties for many students, and this effect is exacerbated in crisis conditions. We discovered that programmed learning technology is effective in crisis situations. In accordance with the technology of programmed learning, the amount of educational material is clearly defined and, if possible, minimized. Basic concepts and facts are highlighted in each topic. The educational material is divided into distinct parts, which are arranged in a clear and logical sequence. Precise and specific instructions for completing tasks necessary for learning each portion of the material are formulated. The assimilation of each part of the educational material is monitored with the help of self-assessment questions and teacher checks.

Elements of programmed learning technology are effective not only in martial law or other crisis conditions. This technology enables the organization of independent learning for higher mathematics students in general or for individual course topics.

Learning takes place in a distance (partially blended) form in crisis conditions.

It is effective to use elements of the «flipped classroom» technology. For studying each topic in the mathematics course, we employed the following scheme: a practical lesson, a lecture, and a subsequent practical lesson. The educational material, which students review in accordance with school textbooks (questions for study are provided in advance), is reviewed during the first practical session. New concepts of the given topic are introduced after reviewing the known facts from the lecture, which are then worked out and systematized in the second practical session.

Elements of the “flipped classroom” technology, the effectiveness of which we tested in the process of teaching a higher mathematics course under martial law, can be used by teachers of various disciplines when teaching “cross-cutting”

topics or when implementing interdisciplinary connections in educational material.

To improve the quality of assimilation of mathematical knowledge in crisis conditions, it is advisable to clearly formulate tasks, consider tasks of the main types, and provide examples of solving such tasks.

In the process of teaching mathematics in crisis conditions, we consider the stage of generalization and systematization of educational material to be mandatory.

We believe it is possible to significantly enhance the effectiveness of training by utilizing specialized means of visualizing educational material during the stages of generalization and systematization of knowledge. Two means of visualizing educational material are the most effective for learning mathematics: tables, which help systematize learning material, and schemes, which help establish connections between concepts.

The value of the article for teachers and students studying mathematics in higher education

At the current stage of societal development, the issue of inadequate mathematical training for future education specialists is a pressing concern for many countries worldwide. We hope that our experience in teaching mathematics at Sumy National Agrarian University (Ukraine) will be of interest to all teachers and students studying mathematics in higher education. In the article, we highlighted the problems that may arise in any educational institution in crisis conditions. We have described methodical ways of solving them. These ways improve the quality of mathematical training of future specialists in various areas and can be implemented in crisis conditions (also in the usual, normal educational process).

We have become convinced that in any crisis conditions (even during the war), it is necessary to find opportunities for schools and universities to work! This has a positive impact on the emotional stability of students, university teachers, and society as a whole.

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